## A geometrical investigation using GeoGebra

- Toolbar selections are shown in <Red>
- Icon selections are shown in <Blue>

Instructions
(1). Toolbar:

Make sure that only the following options are selected (You will see a tick next to them if they are)

```
<View }->\mathrm{ Algebra>
<View }->\mathrm{ Graphics>
<View }->\mathrm{ Graphics2>
```

(2).


At the top of the $1^{\text {st }}$ Graphics Window, click on the small icon to show/reveal the "Quick" Graphics Window sub-Toolbar. Within this, click on the <Axes Icon> $\downarrow$ to clear the Graphics Window of its axes - if present.
(3). Toolbar:
<Options $\rightarrow$ Labelling $\rightarrow$ Automatic>

This should be the default option
(4). Icon Bar: (3rd Icon set)

## <Segment between Two Points>

Draw a segment $A B$ about 4cm long in the centre of the Graphics Window
(5). Icon Bar: (2nd Icon set)

```
<New Point>
```

Click on segment $A B$ to place a movable point on to it - Drag the new point, $C$, to check that it is restricted to movement along segment $A B$ only.
(6). Icon Bar: (6 th Icon set)
<Circle with Centre through Point>

Construct a circle having centre Point B, and radius BA, the length of your original segment.
(7). Icon Bar: (6 ${ }^{\text {th }}$ Icon set)
<Circle with Centre through Point>

Construct a second circle having centre Point $B$, and radius $B C, C$ being the movable point on your original segment. You should have a picture which looks something like this shown to the right here.


Notice: If you wish to remove the labels from any objects then right click on the object and from the context-menu that appears make sure that the "Show Label" option is not highlighted.
(8). Icon Bar: (2 ${ }^{\text {nd }}$ Icon set)
<New Point>

Place a new point on each of your two circles, points $D$ and $E$, say.
(9). Icon Bar: (4 $4^{\text {th }}$ Icon set)

## <Polygon>

Click on the points in the following order to construct a triangle through them: $B-D-E-B$.
(10). Icon Bar: (3rd Icon set)
<Line through Two Points>
Draw a line (Not a segment) through the points $D$ and $E$.
(11). Icon bar: (4 $4^{\text {th }}$ Icon set)

## <Perpendicular Line>

Click on the point $B$, the centre of your circles, then click on the line created in (10) above. This should create a new line through point $B$.
(12). Icon bar: (2nd Icon set)

## <Intersect Two Objects>

Click on the perpendicular line through point $B$, then click on the line created in (10) above. This should create their common point of intersection, a point $F$.

Your diagram may look something like this picture to the left below. Your next task is to right click on objects, choose Object Properties from the context-menu that appears and find out how to hide, colour, change line styles, etc., so as to make your construction look more like one in the picture to the right below.


You will notice that in addition to your modifications of existing objects two more segments need to be added - the red perpendicular height of the triangle and its blue base.
(13). When objects are created in GeoGebra they are often defined in the Algebra View together with an associated measurement.
For example, segments have their lengths, polygons have their area (not perimeter, though this is easily found) and their given named takes this value if used in any later calculations or formulae.

(14). Right-click on your triangle and from the context-menu click on Rename. Then from the Rename dialogue box re-name your polygon "Area"
(15). Toolbar:

$$
\text { <View } \rightarrow \text { Graphics2> }
$$

This will open a second Graphics Window in which constructions can be drawn. Next, click on the Graphics2 Window to activate it, then in the Input Box at the bottom of the screen type the following: $P=(h$, Area). Input: $P=(h$, are
You should then see a point, $P$,


For some orientations of the triangle when its height is not directly above its base the red perpendicular seems "unconnected". Therefore connect a dotted segment to its foot and the base of the triangle created in the Graphics2
Window. Move one of the vertices of your triangle BDE in the first Graphics View to see what effect it has on the point $P$.
(16). Click on the second Graphics Window to activate it and then in the Input Box type: Locus[P, D].
This will create the path or locus that $P$ takes as the point $D$ is moved on your triangle in the first Graphics Window. With suitable modifications it may look like the following:

(17). Icon bar: ( $11^{\text {th }}$ Icon set)
<Check Box to Show/Hide Objects>

Click in the Graphics2 Window and from the Check Box dialog box give it the name:

## "Perp-Area"

Whilst the dialog box is still open click on the objects you wish to hide, namely the locus line of $P$ and the point $P$. Then click on the "Apply" Button to close it.

Clicking on the embedded Check Box will now hide these
 two objects.
(18). Repeat the steps (15) to (17) but this time for the locus of the point connecting the length of the base of the triangle with the area of the triangle.


Note that, if an object that you hoped would appear in the $2^{\text {nd }}$ Graphics Window appears instead in the $1^{\text {st }}$ Graphics Window then right-click on the object and from the Advanced Tab in its Properties dialog box remove the tick in the "Graphics" Check Box and put a tick in the "Graphics 2" Check Box in the "Location" section

Now alter the sizes of your original circles and vary the points of the triangle and write down some observations.

What could you conjecture about the relation between the perimeter of the triangle and its perpendicular height or its base as the triangle's shape is varied?

What could you conjecture about the relation between the area and the perimeter of the triangle as its shape is varied?

